Appendix B: Three-choice "Wallflower" model proofs

Here we consider a simplified three-choice version of our model. Agents choose between contribution level $a \in \{0,1,2\}$ where $c(a) = ka^2$. Agents are one of two types: $g \in \{W,H\}$, where W denotes "wallflower" types and H denotes "honor-seeking." The utility function is:

$$u(a|g,x,v) = va - c(a) + xR(a|g) \tag{B.1}$$

where

$$R(a|g) = h(g)max[E(v|a,g) - \bar{v}, 0] - max[\bar{v} - E(v|a,g), 0]$$
$$h(g) = 1 \; when \; g = H \; and \; h(g) = -1 \; when \; g = W \tag{B.2}$$

With uniform distribution, the expected type of an individual who gives a is the midpoint of the cutoff types for a and $a+1$:

$$E(v|0,g,x) = \frac{\tilde{v}_1^g}{2}$$
$$E(v|1,g,x) = \frac{\tilde{v}_1^g + \tilde{v}_2^g}{2}$$
$$E(v|2,g,x) = \frac{\tilde{v}_2^g + A}{2} \tag{B.3}$$

Since $E(v|0,g,x) \leq \bar{v}$ and $E(v|2,g,x) \geq \bar{v}$, we can write $R(0|g)$ as $\frac{\tilde{v}_1^g - A}{2}$ and $R(2|g)$ as $\frac{\tilde{v}_2^g}{2}$ for honor-seeking types and $-\frac{\tilde{v}_2^g}{2}$ for wallflowers.

Define marginal reputation benefit as $r(a|g) \equiv R(a|g) - R(a-1|g)$. For honor-seeking types, $R(1|H) = \frac{\tilde{v}_1^H + \tilde{v}_2^H - A}{2}$ whether $\tilde{v}_1^H$ is less than or larger than $\bar{v}$. Honor-seeking marginal reputation for contributing a higher amount is always positive.

$$r(1|H) = \frac{\tilde{v}_2^H}{2} > 0$$
$$r(2|H) = \frac{A - \tilde{v}_1^H}{2} > 0 \tag{B.4}$$

For wallflowers $R(1|W)$ depends on whether $E(v|1,W,x)$ is above and below $\bar{v}$, the average type. Consider the first case. $E(v|1,W,x) > \bar{v}$ implies $R(1|W) = \frac{A - \tilde{v}_1^W - \tilde{v}_2^W}{2}$. Here, since $a = 1$ already signals a type above the average type, the marginal reputation benefit of increasing contribution to $a = 2$ only further intensifies (unwanted) image signals ($r(2|W) < 0$).

$$r(1|W) = A - \tilde{v}_1^W - \frac{\tilde{v}_2^W}{2}$$
$$r(2|W) = \frac{\tilde{v}_1^W - A}{2} < 0 \tag{B.5}$$

Now consider the latter case. When $E(v|1,W,x) < \bar{v}$, $R(1|W) = \frac{\tilde{v}_1^W + \tilde{v}_2^W - A}{2}$. Here, since $a = 1$ already signals a type below the average type, decreasing contribution from $a = 1$ to $a = 0$ will further intensifies stigma.

For those contributing $a = 0$, the marginal reputation benefit of increasing contribution to $a = 1$ is therefore positive ($r(1|W) > 0$).

$$r(1|W) = \frac{\tilde{v}_2^W}{2} > 0$$
$$r(2|W) = \frac{A - \tilde{v}_1^W}{2} - \tilde{v}_2^W \tag{B.6}$$

Lemma 1 $A \geq c_2 \Rightarrow E(v|a = 1, x = 1) \leq \bar{v}$

Proof. Suppose $A \geq c_2$ and $E(v|a = 1) > \bar{v}$. Then using the definition of cutoff types: $v_a^g = c(a) - c(a-1) - xr(a|g)$ and Eq. B.5, we solve for wallflower cutoff types for visible contributions:

$$\tilde{v}_1^W = c_1 - r(1|W) = c_1 - A + \tilde{v}_1^W + \frac{\tilde{v}_2^W}{2}$$

$$\tilde{v}_2^W = c_2 - c_1 - r(2|W) = c_2 - c_1 - \frac{\tilde{v}_1^W - A}{2}$$

The first equation implies

$$\tilde{v}_2^W = 2(A - c_1) \tag{B.7}$$

Substituting this to the second equation above, we arrive at

$$\tilde{v}_1^W = 2(c_1 + c_2 - \frac{3}{2}A) \tag{B.8}$$

Substituting $\tilde{v}_1^W$ and $\tilde{v}_2^W$ into our assumption that $E(v|a = 1) = \frac{\tilde{v}_1^g + \tilde{v}_2^g}{2} > \bar{v} = \frac{A}{2}$, we arrive at

$$c_1 + c_2 - \frac{3}{2}A + A - c_1 > \frac{A}{2}$$

The above equation implies $A < c_2$ which is a contradiction. □

Lemma 2 When contributions are visible, the cutoffs types are:

$$\tilde{v}_1^H = \frac{2}{5}(\frac{A}{2} - c_2 + 3c_1) \qquad \tilde{v}_1^W = A - 2c_2 + 2c_1$$
$$\tilde{v}_2^H = \frac{2}{5}(2c_2 - c_1 - A) \qquad \tilde{v}_2^W = 2(2c_2 - c_1 - A) \tag{B.9}$$

Proof. When contributions are visible $\tilde{v}_2^g = c(a) - c(a-1) - r(a|g)$. By Lemma 1 we know that $A \geq c_2 \Rightarrow E(v|a = 1, x = 1) \leq \bar{v}$, which means we only have to be concerned with wallflower reputation as defined by Eq. B.6. Since $r(1|g) = \frac{\tilde{v}_2^g}{2}$ for both gender:

$$\tilde{v}_1^g = c_1 - r(1|g) = c_{1-}\frac{\tilde{v}_2^g}{2}$$
$$\tilde{v}_2^g = 2(c_1 - \tilde{v}_1^g) \tag{B.10}$$

However, $r(2|g)$ is gender specific. For honor-seeking types this is:

$$\tilde{v}_2^H = c_2 - c_1 - r(2|H) = c_2 - c_1 - \frac{A - \tilde{v}_1^H}{2} \tag{B.11}$$

Setting Eq. B.10 equal to Eq. B.11, we get $\tilde{v}_1^H = \frac{2}{5}(\frac{A}{2} - c_2 + 3c_1)$, which we substitute back to Eq. B.10 to arrive at $\tilde{v}_2^H$.
For wallflowers:

$$\tilde{v}_2^W = c_2 - c_1 - r(2|W) = c_2 - c_1 - \frac{A - \tilde{v}_1^W}{2} + \tilde{v}_2^W \tag{B.12}$$

From Eq. B.12 we arrive at $\tilde{v}_1^W = A + 2c_1 - 2c_2$ which we substitute to Eq. B.10 to arrive at $\tilde{v}_2^W$. □

With this, we can write Theorem 1, which describes the impact of visibility on the ordering of cutoff types.

Theorem 1

(i) When contributions are not visible, $v_1 = v_1^H = v_1^W = c_1$, and $v_2 = v_2^H = v_2^W = c_2 - c_1$

(ii) When contributions are visible, the cutoffs are well behaved when A is not to large relative to costs of contribution:

$$\tilde{v}_1^H < \tilde{v}_2^H \textit{ and } \tilde{v}_1^W < \tilde{v}_2^W \Leftrightarrow A < 2c_2 - \frac{8}{3}c_1 \tag{B.13}$$

(iii) Within this range, $\tilde{v}_1^W < \tilde{v}_1^H < v_1 < \tilde{v}_2^H < \bar{v} < v_2 < \tilde{v}_2^W$

Proof. (i) Since $xr(a|g) = 0$ for when contributions are not visible, $v_a^g = c(a) - c(a-1)$. This means $v_1^H = v_1^W = c_1$, and $v_2^H = v_2^W = c_2 - c_1$.

(ii) Cutoff types are well behaved when $\tilde{v}_1^g < \tilde{v}_2^g$. Substituting cutoffs for honor-seeking types from Lemma 2:

$$\tilde{v}_1^H = \frac{2}{5}(\frac{A}{2} - c_2 + 3c_1) < \tilde{v}_2^H = \frac{2}{5}(2c_2 - c_1 - A)$$

$$\frac{A}{2} - c_2 + 3c_1 < 2c_2 - c_1 - A$$

Hence we arrive at the following condition:

$$A < 2c_2 - \frac{8}{3}c_1 \Rightarrow \tilde{v}_1^H < \tilde{v}_2^H \tag{B.14}$$

Substituting cutoffs for wallflowers from Lemma 2 we see that the condition where cutoff types are well behaved for wallflower contributors is satisfied automatically when Eq. B.14 is satisfied:

$$\tilde{v}_1^W = A - 2c_2 + 2c_1 < \tilde{v}_2^W = 2(2c_2 - c_1 - A)$$

$$3A < 2(2c_2 - c_1) + 2c_2 - 2c_1$$

$$A < 2c_2 - \frac{4}{3}c_1$$

(iii) We restrict our attention to $c_2 < A < 2c_2 - \frac{8}{3}c_1$ where cutoffs are well behaved. First note that $v_1 < \bar{v} = \frac{A}{2}$ since $c_1 < \frac{c_2}{2}$ by assumption and that $\bar{v} < v_2$ since $2c_2 - \frac{8}{3}c_1 < 2(c_2 - c_1)$.
We now compare the cutoff type for $a = 2$. Lemma 2 directly shows that $\tilde{v}_2^H < \tilde{v}_2^W$ so we will show that $\tilde{v}_2^H < \frac{A}{2}$ and $v_2 < \tilde{v}_2^W$. Starting from the first inequality:

$$\tilde{v}_2^H = \frac{2}{5}(2c_2 - c_1 - A) < \frac{c_2}{2} < \frac{A}{2}$$

$$2c_2 - c_1 - A < \frac{5c_2}{4}$$

$$\frac{3}{4}c_2 - c_1 < A$$

which is true since $A > c_2$. Note also that $\tilde{v}_2^H > 0$ since $A < 2c_2 - \frac{8}{3}c_1$. Now turning to the latter, since $\tilde{v}_2^W$ decrease in A, we substitute the upper bound of A to the inequality below:

$$c_2 - c_1 = v_2 < \tilde{v}_2^W = 2(2c_2 - c_1 - A)$$

$$\frac{c_2}{2} - \frac{c_1}{2} < 2c_2 - c_1 - 2c_2 + \frac{8}{3}c_1$$

$$c_2 < \frac{19}{3}c_1$$

which is true for since $c_2 = 4c_1$
Lastly we compare cutoffs for $a = 1$. Since $\tilde{v}_2^H > 0$, by Eq. B.10, $\tilde{v}_1^H > v_1^H$. We only need to show that $\tilde{v}_1^W < \tilde{v}_1^H$

$$\tilde{v}_1^W = A - 2c_2 + 2c_1 < \tilde{v}_1^H = \frac{2}{5}(\frac{A}{2} - c_2 + 3c_1)$$

$$\frac{4}{5}A < \frac{2}{5}(-c_2 + 3c_1) + 2c_2 - 2c_1 = \frac{4}{5}(2c_2 - c_1)$$

which is true when Eq. B.14 is satisfied.

□

Appendix C: Incentive compatibility of beliefs

In the belief elicitation task, participants guess how many people chose each particular contribution level. Let g_c denote a participant's reported guess as to how many people chose contribution level $c \in \{1,...C\}$. Let n_c denote the actual number of participants who chose that contribution level. Participants' guesses are constrained such that $0 \leq g_c \leq N$ and $\sum_c g_c = N$, where N is the number of other participants in the room. Payoffs are given by $\pi(g) = 0.5 \sum_c min\{g_c, n_c\}$.

Note that because participants have no information about other participants' actions when they make their guesses, their beliefs over the set of all outcomes $\{n_1,...,n_C\}$ are represented by N i.i.d. draws from a multinomial distribution. Let $\{p_1,...,p_C\}$ denote the probabilities of each independent event c that define the multinomial distribution, with $0 \leq p_c \leq N$ and $\sum_c p_c = 1$.

Given these parameters, denote the expected outcome as

$$\{y_1,...,y_C\} = E[\{n_1,...,n_C\}|\{N;p_1,...,p_C\}] \tag{C.1}$$

Note that $\{y_1,...,y_C\} = E[\{n_1,...,n_C\}|\{N;p_1,...,p_C\}] \Rightarrow \{y_1,...,y_C\} = \{Np_1,...,Np_C\}$.

Claim 1

If $\{y_1,...,y_C\} = E[\{n_1,...,n_C\}|\{N;p_1,...,p_C\}]$ and $\{n_1,...,n_C\}$ is generated according to a multinomial distribution defined by parameters $\{N;p_1,...,p_C\}$, then

$$\{y_1,...,y_C\} = argmax_g E[0.5 \sum_c min\{g_c, n_c\}]$$

Proof. We will prove by contradiction. Suppose that $\{y_1,...,y_C\} \neq argmax_g E[0.5 \sum_c min\{g_c, n_c\}]$. This implies that there is some rearrangement of guesses that would increase the participant's expected payoff. More specifically, there exists at least one pair of contribution choices c and c' such that the vector of guesses $\hat{y} = \{y_1,...,y_c+1, y_{c'}-1,...,y_C\}$ yields a higher expected payoff than $\bar{y} = \{y_1,...,y_c,y_{c'},...,y_C\}$. Without loss of generality, we let $c = 1$ and $c' = 2$.

Note that $\{y_1,...,y_C\} = E[\{n_1,...,n_C\}|\{N;p_1,...,p_C\}] \Rightarrow \{y_1,...,y_C\} = \{Np_1,...,Np_C\}$.

Thus, $\bar{y} = \{Np_1, Np_2, Np_3,...,Np_C\}$ and $\hat{y} = \{Np_1+1, Np_2-1, Np_3,...,Np_C\}$.

The expected payoff $E[0.5 \sum_c min\{g_c, n_c\}]$ can be rewritten as:

$$\sum_c g_c(1-p(n_c \leq g_c)) + E[n_c|n_c \leq g_c]p(n_c \leq g_c) = \sum_c g_c(1-F(g_c)) + E[n_c|n_c \leq g_c]F(g_c) \tag{C.2}$$

The marginal distribution of a single component of a multinomial is simply a binomial distribution, so $F(g_c)$ is the binomial CDF defined by parameters (N, p_c). Next, note that $E[n_c|n_c \leq y_c] * F(y_c) = [\sum_0^{y_c} \frac{x*f(x)}{F(y_c)}] * F(y_c) = \sum_0^{y_c} x * f(x)$. Thus, the expected payoff can be further simplified:

$$\sum_c [g_c(1-F(g_c)) + \sum_0^{y_c} x * f(x)] \tag{C.3}$$

The assumption $E\pi(\hat{y}) > E\pi(\bar{y})$ implies

$$(Np_1+1)(1-F(Np_1+1)) + \sum_{0}^{Np_1+1} x*f(x) + (Np_2-1)(1-F(Np_2-1)) + \sum_{0}^{Np_2-1} x*f(x) \tag{C.4}$$
$$> (Np_1)(1-F(Np_1)) + \sum_{0}^{Np_1} x*f(x) + (Np_2)(1-F(Np_2)) + \sum_{0}^{Np_2} x*f(x)$$

This ultimately simplifies to $p(n_2 \leq Np_2 - 1) > p(n_1 \leq Np_1)$ which are binomial CDFs. When the mean of a binomial distribution is an integer, then the median and mean coincide. Thus, under the simplifying assumption that Np_1 and Np_2 are integers[1]:

- Np_2 is a median $\Rightarrow p(n_2 \leq Np_2 - 1) < 0.5$
- Np_1 is a median $\Rightarrow p(n_1 \leq Np_1) \geq 0.5$
- $\Rightarrow 0.5 > p(n_2 \leq Np_2 - 1) > p(n_1 \leq Np_1) \geq 0.5$, which is a contradiction

Thus, it cannot be the case that an individual has an incentive to deviate from a report of $\{y_1, ..., y_C\} = E[\{n_1, ..., n_C\}|\{N; p_1, ..., p_C\}]$. □

Finally, note that we can transform participants' reports into probabilities. As shown in Claim 1, the mechanism incentives participants to report:

$$\{g_1, ..., g_C\} = E[\{n_1, ..., n_C\}|\{N; p_1, ..., p_C\}] = \{Np_1, ..., Np_C\} \tag{C.5}$$

Thus, $g_c/N = p_c$.

[1]This relationship holds without the integer assumption, but is seen more directly when mean and median coincide.

Appendix D: Laboratory experiment instructions

In the text below, instructions unique to Baseline treatment are contained in square brackets: […]. *Instructions unique to Visibility treatment are contained in curly brackets:* {…}. *All other instructions are identical across treatments.*

Preliminary on-screen instructions

(Note: All of the instructions in this section were displayed on a series of screens at participants' computer terminals and read aloud by the experimenter. The experimenter controlled the pace at which participants progressed through these screens.)

Introduction

This experiment is a study of decision-making. You will receive $5 simply for showing up. You will have an opportunity for additional earnings depending on the decision that you make in two tasks: a giving task and a guessing task. In the giving task, you will be given $10 that you can contribute to charity. In the guessing task you can earn up to $7.

Please do not talk to other participants during the experiment. If at any point you have a question, raise your hand and we will come to you to answer it.

We will first introduce you to the charitable cause that you can donate to.

Description of cause

A dedicated team of local engineering professionals as well as University of Pittsburgh and Carnegie-Mellon students has been working on delivering potable water to the homes of a subsistence farming community living at 3600 meters elevation in the Andes Mountains of Ecuador.

However due to topography, lack of a continuous clean water supply has continue to plague development. The community-owned water source is a natural spring located 1000 feet below the village. Currently, community members rely on a daily 2 hour commute to collect potable water and supplement their needs through a rain catchment system constructed in 2009 by the Engineers Without Borders (EWB)-Pittsburgh team.

The Pittsburgh EWB team has lined up partial funding and community ownership for the infrastructure necessary for the ambitious project of providing a continuous water supply. Construction of the pipeline up the mountain is currently taking place. The last step of the project is to install faucets that directly deliver water to households. Your contribution will fund the cost of purchasing and installing faucets in Tingo Pucara.

As the world continues to grow larger and more complicated, there are people out there still walking hours every day to simply have clean water to drink, cook, and bathe. Help EWB-Pittsburgh cross Tingo Pucara off that list.

Unconditional contribution instructions

You will now have the opportunity to donate to the cause that was just described.

You have been given $10 in an envelope. Before proceeding, please sign the receipt that indicates that you have received $10.

You may contribute as much or as little of this money as you would like to the cause. However, we will ask that you restrict your contribution to $2 increments. That is, you will choose whether to contribute $0, $2, $4, $6, $8, or $10. Any money that you do not contribute is yours to keep.

[For today's experiment, you have been randomly assigned to a group with two other participants in the room. Your group's total contributions will go towards purchasing and installing faucets in Tingo Pucara. You will not know the identity of your group members while deciding, nor will you be able to communicate with them about the decision. Your group has the potential to make a large impact; the average cost of faucets and associated installation is $30, but varies with local conditions.

At the end of today's session, you will leave your donation in its original envelope on your desk. The software will inform you of your group's total donation to Tingo Pucara.]

{ For today's experiment, you have been randomly assigned to a group with two other participants in the room. Your group's total contributions will go towards purchasing and installing faucets in Tingo Pucara. You will not know the identity of your group members while deciding, nor will you be able to communicate with them about the decision. Your group has the potential to make a large impact; the average cost of faucets and associated installation is $30, but varies with local conditions.

At the end of today's session, your group members will learn how much you contributed. In particular, you and your group (one group at a time) will go to a conference room with the experimenter to submit your contributions. While your group is waiting for its turn to go to the conference room, you and your group members will gather at the front of this room to fill out a group contribution slip together, indicating how much each member is donating. Your group members are the only participants who will observe how much you chose to give.

In the conference room, the experimenter will ask you for your group contribution slip. When the experimenter reads out your contribution from the slip, you wil count out your donation in front of your group members and hand it to the experimenter. At the end, the experimenter will inform you of your group's total donation to Tingo Pucara and thank you for your donation. }

In a moment the contribution screen will appear on your computer. Again, you may contribute any multiple of $2 in between $0 and $10. If you have a question, raise your hand and an experimenter will come to you to answer it.

(Note: All remaining instructions were distributed as paper handouts and read aloud by an experimenter.)

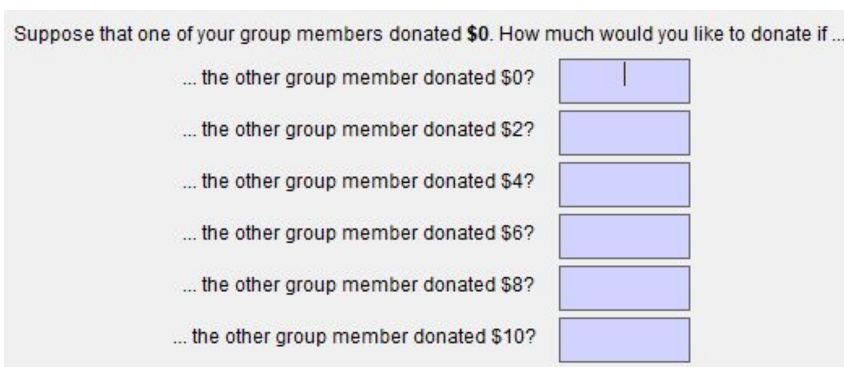
Instructions: Giving Task Part II

We will refer the decision you have just made as your Part I decision.

You may now have an opportunity to change your contribution based on what the other members of your group contributed.

Specifically, in a moment you will indicate what you would have contributed in Part I if you had observed your other two members' contributions. However, because you were not able to observe their contributions, you will be asked to indicate what your decision would have been given every possible pair of contributions.

To do so, you will be presented with a series of screens that capture every potential combination of your other group members' contributions from Part I. For example, the image below is the first screen that you will see; the input boxes in this screen represent all of the scenarios where one of your group members gave $0. Each line indicates the possible contributions of the *other* group member. In each input box, you will indicate the amount you would have contributed if your group members had contributed the amounts associated with that input box.



After you fill in the input boxes in the figure above, the screen below will show up. This screen represents all of the scenarios where one of your group members gave $2. (Notice that it leaves out the situation where one member gives $2 and the other gives $0. This is because you already entered a decision for that situation in the first screen.) For example, the input box labeled "A" is in the "$4" row. Therefore, in that input box you will enter the amount that you would want to contribute if you knew that one member contributed $2 and the other contributed $4.

After this you will see similar screens for all of the scenarios where one member gives $4, $6, $8, and finally $10.

You will fill in *all* of the input boxes on each of these screens. As before, you can enter any multiple of $2 from $0 to $10 in each input box.

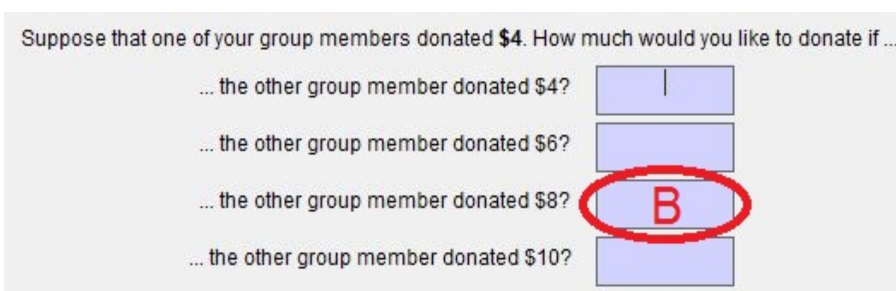
<u>How might these decisions impact the contribution you actually provide?</u>

At the end of the experiment, one member of each group will be randomly selected. If you are that person, the decisions that you make in this phase (Part II) will determine how much you actually contribute to the project.

If you *are not* the randomly selected group member, then the actual amount you will contribute at the end of the experiment will simply be the contribution you indicated in Part I of the giving task.

If you *are* the randomly selected group member, your contribution will be determined by your decision in this phase (Part II) conditional on your group members' contributions from Part I.

For example, suppose that at the end of the experiment you are informed that you are the randomly selected group member. This means that your two group members simply submit their contributions from Part I. Suppose that they chose $4 and $8. Because you are the randomly selected group member, your contribution will be whatever you specify in the input box associated with contributions of $4 and $8 (input box B in figure below.) Suppose you had entered $6 in that box. Then you would submit $6 and keep $4, regardless of the contribution you entered in Part I. Therefore, your group's total contribution would be $18 ($4+$8+$6).



You do not know whether you will be the randomly selected group member when you fill in the contribution tables. You will therefore have to think carefully about these decisions because they may determine how much you contribute to the project.

We are now ready to begin. Before proceeding to the contribution decisions, you will complete a brief quiz. This quiz has no impact on your earnings and is merely intended to ensure that the instructions are clear. Feel free to look back at the instructions while answering the questions. After each question you will be informed of the correct answer.

Instructions: Guessing task

You have now completed the giving task. We will now move to the guessing task, after which you will complete a brief survey. The software will then inform you of the following: 1) whether you were the randomly selected group member 2) your actual contribution to Tingo Pucara and 3) your earnings from the guessing task. This will conclude the session – you will then place your donation as indicated by the computer in its original envelope and leave it on your desk. [The experimenter will then call your experimental ID to pay your show up fee and earnings from the guessing task.] { This will conclude the session – the experimenter will then call each group to submit their donations. }

In the guessing task, you will guess the donation chosen **in Part I** by the other 14 participants in the room. Specifically, you have 14 tokens – one for each participant in the room (not including yourself). There were six possible contribution choices in Part I: $0, $2, $4, $6, $8, or $10. For each of these possible contributions, you will guess how many people in the room chose that contribution and assign your tokens accordingly. For example, if you think that everybody in the room chose $10 then you would assign all 14 tokens to "$10". If you think that half of the people in the room chose $6 and the other half chose $8, then you would assign 7 tokens to "$6" and 7 tokens to "$8."

For each token that you place correctly, you will receive $0.50. You receive nothing for each token that is placed incorrectly. For example, again suppose that you think that half of the participants chose $6 and the other half chose $8, so you assign 7 tokens to each of these. Suppose that instead, two people chose $6 and everyone else in the room chose $2. This means that exactly two of your guesses were correct so you would receive $1.00. If your guess had been correct – that is, if it were actually the case that 7 people chose $6 and 7 people chose $8 – then you would have received $7.

After everyone has completed the guessing task and a short survey, the software will display the information above. When you are informed of your earnings from the guessing task, please indicate this amount and show up fee on your receipt.